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PROPERTIES OF A(z)-ANALYTIC FUNCTIONS

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Abstract

This article presents some properties of A(z)-analytic functions, an analogue of Koshi's theorem for these functions, Koshi's formula, an analogue of Weyershtrass's theorem, and an analogue of Schwartz's Lemma.

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Proposition 1. *The linear combination of analytic functions on a finite number is again A(z) - analytic, if* $f_1(z), f_2(z), ..., f_n(z)$ *are A(z)-analytic functions and* $\alpha_j \in R,\ \ j=1,2,...,n$ *,then the function* $f(z)=\alpha_1 f_1(z)+\alpha_2 f_2(z)+...+\alpha_n f_n(z)$ *is also A(z) - analytic.*

Proposition 2. *A function consisting of a product of two A(z)-analytic functions will also be an A-analytic function, if* $f_1(z),\ f_2(z)$ *are analytic functions, then their product* $f_1(z)\cdot f_2(z)$ *will also be an analytic function.*

Proof. - $f_1(z),\ f_2(z)$ are analytic functions. So that, $\overline{D_A} f_1(z)=0$ and $\overline{D_A} f_2(z)=0$, it is $\frac{\partial f_1(z)}{\partial \overline{z}}=A(z)\cdot\frac{\partial f_1(z)}{\partial z}$ and $\frac{\partial f_2(z)}{\partial \overline{z}}=A(z)\cdot\frac{\partial f_2(z)}{\partial z}$. From these equalities

$$\frac{\partial}{\partial \overline{z}}\left[f_1(z)\cdot f_2(z)\right]=\frac{\partial f_1(z)}{\partial \overline{z}}\cdot f_2(z)+f_1(z)\cdot\frac{\partial f_2(z)}{\partial \overline{z}}=$$

$$= A(z)\cdot\frac{\partial f_1(z)}{\partial z}\cdot f_2(z)+f_1(z)\cdot A(z)\cdot\frac{\partial f_2(z)}{\partial z}=$$

$$= A(z)\cdot\left(\frac{\partial f_1(z)}{\partial z}\cdot f_2(z)+f_1(z)\cdot\frac{\partial f_2(z)}{\partial z}\right)= A(z)\cdot\frac{\partial}{\partial z}\left[f_1(z)\cdot f_2(z)\right]$$

We have that. From this comes the necessary confirmation.

Proposition 3. *The ratio of two A(z)-analytic functions will also be an A (z)-analytic function, if* $f_1(z),\ f_2(z)$ *are A(z)-analytic functions(* $f_2(z)\neq 0$ *), then the function* $\frac{f_1(z)}{f_2(z)}$ *is also A(z)-analytic function.*

Proof. So that functions $f_1(z),\ f_2(z)$ are differentiable functions and it is $f_2(z)\neq 0$, the function $\frac{f_1(z)}{f_2(z)}$ is also differentiable. They are a (z)-analytic functions, so that the equalities $\frac{\partial f_1(z)}{\partial \overline{z}}= A(z)\cdot\frac{\partial f_1(z)}{\partial z}$ and $\frac{\partial f_2(z)}{\partial \overline{z}}= A(z)\cdot\frac{\partial f_2(z)}{\partial z}$ holds, they will be

$$\frac{\partial}{\partial \overline{z}}\left[\frac{f_1(z)}{f_2(z)}\right]=\frac{\frac{\partial f_1(z)}{\partial \overline{z}}\cdot f_2(z)-\frac{\partial f_2(z)}{\partial \overline{z}}\cdot f_1(z)}{\left[f_2(z)\right]^2}=$$

$$=\frac{A(z)\frac{\partial f_1(z)}{\partial z}\cdot f_2(z)-A(z)\frac{\partial f_2(z)}{\partial z}\cdot f_1(z)}{\left[f_2(z)\right]^2}=$$

$$= A(z)\frac{\frac{\partial f_1(z)}{\partial z}\cdot f_2(z)-\frac{\partial f_2(z)}{\partial z}\cdot f_1(z)}{\left[f_2(z)\right]^2}= A(z)\frac{\partial}{\partial z}\left[\frac{f_1(z)}{f_2(z)}\right]$$

and this proves the said confirmation.

Proposition 4. *If the function* $f(z)$ *is A(z) - analytic function, then the function* $\overline{f(z)}$ *is an antianalytic function.*

It should be noted that when we obtain definitions of analytic and antianalytic functions respectively.

In the case of almost everywhere $|A(z)|<1$ in the D sphere, the homeomorph solution does not change the orientation, When at the time it is $|A(z)|>1$ there are such subsets of D, and in one it changes in almost everywhere. In these cases, the Beltrami equation is formally defined. This case is expressed in an interesting way, when at the same time there are such sets of parts of D , let in one it be done almost $|A(z)|<1$ everywhere, and in another it is done almost $|A(z)|>1$ everywhere. We say that in this situation, the Beltrami equation will be the type that changes. We describe its set of solutions. The first to consider the issue of researching the type of variation of the Beltrami equation is put by L.I.Volkovsky. In the general case related to the study of the classical Beltrami equation $f_{\overline{z}}= A^*(z)\,f_z(z)$, the equation $f_{\overline{z}}= A(z)\,f_z(z)$

$$A^*(z)=\begin{cases} A(z), & |A(z)|\le 1 \quad da; \\ \dfrac{1}{\overline{A}(z)}, & |A(z)|>1 \quad da \end{cases}$$

we learn together with complex dilation.

Theorem 1. *For function* $A(z):\|A\|_\infty<1$ *in the arbitrary-dimensional C complex plane , there is such a single homeomorph solution* $X(z)$ *of the* $f_{\bar z}=A(z)f_z(z)$ *equation that the solution X excitable points* $0,1,\infty$ *are abandoned.*

It is worth noting that if function $A(z)\ \ (|A(z)|\le c<1)$ is defined only in the domain $D\subset C$, then it can be continued to the whole complex plane C, assuming that it is $A\equiv 0$ outside the domain D, let our theorem be true for the arbitrary domain $D\subset C$.

Theorem 2. *It is defined by* $f(z)=\hat{O}(X(z))$ *the set of all general solutions of the equation* $f_{\bar z}=A(z)f_z(z)$, $X(z)-$ *the homeomorph solution in the theorem,* $F(\zeta)-X(D)$ *holomorph function on* ζ *. In addition, the holomorphic function* $F=f\circ X^{-1}$ *switches to the specific to* f *with the stored type.*

It follows from theorem 2.2.2, that A(z)-analytic function f performs internal reflections, that is, it reflects an open set to an open set. From here follows the correctness of the maximal principle: for an arbitrary bounded domain $D\subset J$, the module achieves to maximum only on the boundary:

$$|f(z)|<\max_{z\in\partial D}|f(z)|,\ z\in D.$$

If the function does not become zero, then the minimum principle holds:

$$|f(z)|>\min_{z\in\partial D}|f(z)|,\ z\in D.$$

`**Theorem 3.** *If the function* $A(z)$ *belongs to the class of times m differentiable functions:* $A(z)\in C^m(D),$ *then the arbitrary solution* f *of the equation* $f_{\bar z}=A(z)f_z(z)$*, like the minimum, belongs to this class, so* $f\in C^m(D).$

Theorem 4 (an analogue of Koshi's theorem). *If* $f(z)\in O_A(D)\cap C(\overline{D}),$ *there is an area* ∂D *in which the limit is rectifiable, then it is appropriated* $\int_{\partial D} f(z)(dz+A(z)d\bar z)=0$*. When we study A(z)-analytic function, the kernel is of great importance in being A(z)-antianalytic function:*

$$K(z;\zeta)=\frac{1}{2\pi i}\frac{1}{z-\zeta+\int\limits_{\gamma(z;\zeta)}\overline{A(\tau)}d\tau} \qquad (2.2.1)$$

here, we mark $\gamma(z;\zeta)-\zeta,\ z\in D$ *the points with* $\gamma(z;\zeta)$ *a connecting curve or an arbitrary path from point* ζ *to point z.*

Thus a field D is a one-link and $\overline{A}(z)-$holomorphic function, then $I(z)=\int\limits_{\gamma(\zeta;z)}\overline{A}(\tau)d\tau$ is not depended on the path of integration, which the initial state coincides with $I'(z)=\overline{A}(z)$. If there is a field $D\subset C$ then the following theorem holds.

Theorem 5. *The kernel* $K(z;\zeta)$ *is A(z)-analytic function at points z=ζ other than z=ζ, so* $K\in O_A(D\setminus\zeta)$. *Also, the point* $z=\zeta$ *for* $K(z;\zeta)$ *is the first ordered polar point.*

Lemma (an analogue of Schwartz's Lemma). *They are* $f\in O_A(L(a;R)),\ |f(z)|\le M$ *and* $f(a)=0$. *Then it is appropriate* $|f(z)|\le\frac{M}{R}|\psi(z;a)|$ (2.2.5.) *for all* $z\in L(a;R)$.

Proof. If it is $f(a)=0$, then $g(z)=\frac{f(z)}{\psi(z;a)}\in O_A(L(a;R)),\ r<R$ fixed. From the maximal principle, the function $g(z)$ achieves its maximum on $\partial L(a;r)$. Then

$$|g(z)|\le\frac{max|f(z)|:z\in L(a;r)}{|\psi(z;a)|}\le\frac{M}{r}$$

when we strive we move to the limit $r\to R$, that is $|g(z)|\le\frac{M}{R},\ |f(z)|\le\frac{M}{R}|\psi(z;a)|,$ for all $z\in L(a;r)$ and for all $r<R$. The proof is over.

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